JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 22nd JANUARY 2025)

TIME : 3 : 00 PM TO 6 : 00 PM

MATHEMATICS

SECTION-A

1. Let α, β, γ and δ be the coefficients of x^7, x^5, x^3 and x respectively in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $x > 1$. If u and v satisfy the equations

$\alpha u + \beta v = 18$,

$\gamma u + \delta v = 20$,

then u + v equals :

(1) 5 (2) 4

(3) 3 (4) 8

Ans. **(1)**

Sol. $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$= 2\{{}^5C_0.x^5 + {}^5C_2.x^3(x^3-1) + {}^5C_4.x(x^3-1)^2\}$

$= 2\{5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\}$

$\Rightarrow \alpha = 10, \beta = 2, \gamma = -20, \delta = 10$

Now, $10u + 2v = 18$

$-20u + 10v = 20$

$\Rightarrow u = 1, v = 4$

$u + v = 5$

2. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

(1) 144 (2) 72

(3) 96 (4) 120

Ans. **(1)**

Sol. Total – when B_1 and B_2 are together

$= 2!(3!\ 4!) - 2!\ (3!(3!\ 2!)) = 144$

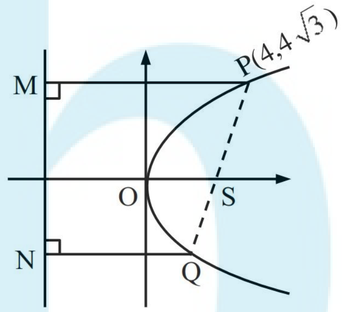
3. Let $P\left(4,4\sqrt{3}\right)$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to:

(1) $\dfrac{263\sqrt{3}}{8}$ (2) $17\sqrt{3}$

(3) $\dfrac{343\sqrt{3}}{8}$ (4) $\dfrac{34\sqrt{3}}{3}$

Ans. **(3)**

Sol.



$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a.4$

$4a = 12$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \dfrac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q is $t_2 = -\dfrac{\sqrt{3}}{2} \Rightarrow Q\left(\dfrac{9}{4},-3\sqrt{3}\right)$

Area of trapezium PQNM

$= \dfrac{1}{2}MN.(PM + QN)$

$= \dfrac{1}{2}MN.(PS + QS)$

$= \dfrac{1}{2}MN.\ PQ$

$= \dfrac{1}{2}7\sqrt{3}.\dfrac{49}{4} = (343)\dfrac{\sqrt{3}}{8} = 3$

4. For a 3 × 3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3 × 3 matrix such that $|A| = \frac{1}{2}$ and trace (A) = 3. If B = adj(adj(2A)), then the value of |B| + trace (B) equals:

(1) 56 (2) 132

(3) 174 (4) 280

Ans. (4)

Sol. $|A|=\frac{1}{2}$, trace(A) = 3, B = adj(adj(2A))=$|2A|^{n-2}(2A)$

n = 3, B = |2A|(2A) = 2^3.|A|(2A) = 8A

|B| = |8A| = 8^3.|A| = 2^8 = 256

trace(B) = 8 trace(A) = 24

|B| + trace(B) = 280

5. Suppose that the number of terms in an A.P. is 2k, k∈N. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to

(1) 5 (2) 8

(3) 6 (4) 4

Ans. (1)

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{2k} \rightarrow$ A.P.

$$\sum_{r=1}^{k} a_{2r-1} = 40,\ \sum_{r=1}^{k} a_{2r} = 55,\ a_{2k} - a_1 = 27$$

$$\frac{k}{2}[2a_1 + (k-1)2d] = 40,\ \frac{k}{2}[2a_2 + (k-1)2d] = 55,$$

$$d = \frac{27}{2k-1}$$

$$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$$

$$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9k = 10k - 5$$

∴ k = 5

6. Let a line pass through two distinct points P(–2, –1, 3) and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point R(1, 3, 3) is 5, then the square of the area of ΔPQR is equal to:

(1) 136 (2) 140

(3) 144 (4) 148

Ans. (1)

Sol. $\overrightarrow{PQ}$ parallel to $3\hat{i}+2\hat{j}+2\hat{k}$, R(1, 3, 3)

$\Rightarrow Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3),\ \lambda \in R - \{0\}$

$$|\overrightarrow{QR}| = 5 = \sqrt{(3\lambda-3)^2 + (2\lambda-4)^2 + (2\lambda)^2}$$

$\therefore 17\lambda^2 - 34\lambda + 25 = 25 \Rightarrow \lambda = 2 (\because \lambda \neq 0)$

∴ Q(4, 3, 7), P(–2, –1, 3), R(1, 3, 3)

$$\text{Area of } \Delta PQR = [PQR] = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$[PQR] = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{matrix}\right\| = \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & 4 & 0 \end{matrix}\right\|$$

$$[PQR] = |-8\hat{i} + 6\hat{j} + 6\hat{k}| = \sqrt{136}$$

$\therefore [PQR]^2 = 136$

7. If $\lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right)^x = \alpha$, then the value of $\frac{\log_e \alpha}{1+\log_e \alpha}$ equals :

(1) e (2) e^{-2}

(3) e^2 (4) e^{-1}

Ans. (1)

Sol. $\alpha = \lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right)^x$ (1^∞ form)

$\therefore \alpha = e^L$

$$\text{Where } L = \lim_{x\to\infty} x\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right) - 1\right)$$

$$\Rightarrow L = \lim_{x\to\infty}\left(\frac{e}{1-e}\right)x\left(\frac{1}{e} - \frac{x}{1+x} - \left(\frac{1-e}{e}\right)\right)$$

$$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty} x\left(1 - \frac{x}{1+x}\right)$$

$$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty}\frac{x}{x+1}$$

$$\Rightarrow L = \frac{e}{1-e}.1$$

$$\Rightarrow L = \frac{e}{1-e}$$

$$\therefore \alpha = e^{\frac{e}{1-e}} \Rightarrow \log\alpha = \frac{e}{1-e}$$

$$\therefore \text{Required value} = \frac{\frac{e}{1-e}}{1 + \frac{e}{1-e}} = e$$

8. Let $f(x) = \int_0^{x^2} \frac{t^2 - 8t + 15}{e^t} dt$, $x \in \mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are :

(1) 2 and 3　　(2) 3 and 2

(3) 1 and 3　　(4) 2 and 2

Ans. **(1)**

Sol. $f'(x) = \left(\frac{x^4 - 8x^2 + 15}{e^{x^2}}\right)(2x)$

$= \frac{(x^2-3)(x^2-5)(2x)}{e^{x^2}}$

$= \frac{(x-\sqrt{3})(x+\sqrt{3})(x-\sqrt{5})(x+\sqrt{5})2x}{e^{x^2}}$

$$\begin{array}{ccccccccccc} - & & + & & - & & + & & - & & + \\ & -\sqrt{5} & & -\sqrt{3} & & 0 & & \sqrt{3} & & \sqrt{5} & \end{array}$$

Maxima at $x \in \{-\sqrt{3}, \sqrt{3}\}$

Minima at $x \in \{-\sqrt{5}, 0, \sqrt{5}\}$

2 points of maxima and 3 points of minima.

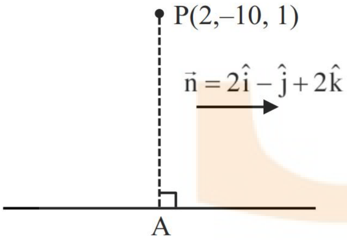
9. The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point P(2, –10,1), is:

(1) 6　　(2) $5\sqrt{2}$

(3) $3\sqrt{5}$　　(4) $4\sqrt{3}$

Ans. **(3)**

Sol.



$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2} = \lambda$ (let)

$(2\lambda + 1, -\lambda - 2, 2\lambda - 3)$

$\because \overrightarrow{PA}.\vec{n} = 0$

$\Rightarrow (2\lambda - 1)2 + (-\lambda + 8)(-1) + (2\lambda - 4)2 = 0$

$\Rightarrow 4\lambda - 2 + \lambda - 8 + 4\lambda - 8 = 0$

$\Rightarrow 9\lambda - 18 = 0 \Rightarrow \lambda = 2$

$\therefore A(5, -4, 1)$

$\therefore AP = \sqrt{3^2 + 6^2 + 0^2} = \sqrt{45} = 3\sqrt{5}$

10. If x = f(y) is the solution of the differential equation

$(1 + y^2) + \left(x - 2e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

with f(0) = 1, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(1) $e^{\pi/4}$　　(2) $e^{\pi/12}$

(3) $e^{\pi/3}$　　(4) $e^{\pi/6}$

Ans. **(4)**

Sol. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{2e^{\tan^{-1} y}}{1+y^2}$

I.F. $= e^{\tan^{-1} y}$

$xe^{\tan^{-1} y} = \int \frac{2(e^{\tan^{-1} y})^2 dy}{1+y^2}$

Put $\tan^{-1} y = t$, $\frac{dy}{1+y^2} = dt$

$xe^{\tan^{-1} y} = \int 2e^{2t} dt$

$xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$

$x = e^{\tan^{-1} y} + ce^{-\tan^{-1} y}$

$\because y = 0, x = 1$

$1 = 1 + c \Rightarrow c = 0$

$y = \frac{1}{\sqrt{3}}$, $x = e^{\pi/6}$

11. If $\int e^x \left(\frac{x\sin^{-1} x}{\sqrt{1-x^2}} + \frac{\sin^{-1} x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right) dx = g(x) + C$,

where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(1) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$　　(2) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$

(3) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$　　(4) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$

Ans. **(3)**

Sol. $\because \frac{d}{dx}\left(\frac{x\sin^{-1} x}{\sqrt{1-x^2}}\right) = \frac{\sin^{-1} x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}$

$\Rightarrow \int e^x \left(\frac{x\sin^{-1} x}{\sqrt{1-x^2}} + \frac{\sin^{-1} x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right)dx$

$= e^x . \frac{x\sin^{-1} x}{\sqrt{1-x^2}} + c = g(x) + C$

Note : assuming $g(x) = \frac{xe^x \sin^{-1} x}{\sqrt{1-x^2}}$

$g(1/2) = \frac{e^{1/2}}{2} . \frac{\frac{\pi}{6} \times 2}{\sqrt{3}} = \frac{\pi}{6}\sqrt{\frac{e}{3}}$

Comment : In this question we will not get a unique function g(x), but in order to match the answer we will have to assume $g(x) = \frac{xe^x \sin^{-1} x}{\sqrt{1-x^2}}$.

12. Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0$, $\theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then 16(M + m) equals :

(1) 24 (2) 25

(3) 27 (4) 17

Ans. (2)

Sol. $(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$$\left[\frac{\cos^2\theta}{4}+1\right]^2 - 2.\frac{1}{4}$$

$$\left(\frac{\cos^2\theta}{4}+1\right)^2 - \frac{1}{2}$$

$$M = \frac{25}{16} - \frac{1}{2} = \frac{17}{16}$$

$$m = \frac{1}{2},\ 16(M+m) = 25$$

13. Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16}. Then the number of many-one functions $f : A \to B$ such that $1 \in f(A)$ is equal to :

(1) 127 (2) 151

(3) 163 (4) 139

Ans. (2)

Sol. Total = 4^4

One-one = 4!

Many-one = 256 – 24 = 232

Many-one which $1 \notin f(A)$

= 3.3.3.3 = 81

232 – 81 = 151

14. If the system of linear equations :

$x + y + 2z = 6,$

$2x + 3y + az = a + 1,$

$-x - 3y + bz = 2b,$

where a, b ∈ **R**, has infinitely many solutions, then 7a + 3b is equal to :

(1) 9 (2) 12

(3) 16 (4) 22

Ans. (3)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{vmatrix} = 0$

$\Rightarrow 2a + b - 6 = 0$ ....(1)

$\Delta_1 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & a+1 \\ -1 & -3 & 2b \end{vmatrix} = 0$

$\Rightarrow a + b - 8 = 0$ ....(2)

Solving (1) + (2)

a = –2, b = 10

$\Rightarrow 7a + 3b = 16$

15. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in [–1, 3] is :

(1) 3 (2) 2

(3) 1 (4) 0

Ans. (4)

Sol. $\hat{a}.\hat{b} = \frac{1}{2}$

Now $(\lambda\hat{a} + 2\hat{b}).(3\hat{a} - \lambda\hat{b}) = 0$

$3\lambda\hat{a}.\hat{a} - \lambda^2\hat{a}.\hat{b} + 6\hat{a}.\hat{b} - 2\lambda\hat{b}.\hat{b} = 0$

$$3\lambda - \frac{\lambda^2}{2} + 3 - 2\lambda = 0$$

$\lambda^2 - 2\lambda - 6 = 0$

$\lambda = 1 \pm \sqrt{7}$

$\Rightarrow$ number of values = 0

16. Let E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b and H : $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If a – A = 2, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to :

(1) 10 (2) 7

(3) 8 (4) 9

Ans. (3)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ foci are (ae, 0) and (–ae, 0)

$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ foci are (Ae′, 0) and (–Ae′, 0)

$\Rightarrow 2ae = 2\sqrt{3} \Rightarrow ae = \sqrt{3}$

and $2Ae' = 2\sqrt{3} \Rightarrow Ae' = \sqrt{3}$

$\Rightarrow ae = Ae' \Rightarrow \frac{e}{e'}=\frac{A}{a}$

$\Rightarrow \frac{1}{3}=\frac{A}{a} \quad \Rightarrow a = 3A$

Now $a - A = 2 \Rightarrow a - \frac{a}{3} - 2 \Rightarrow a = 3$ and $A = 1$

$Ae = \sqrt{3} \Rightarrow e = \frac{1}{\sqrt{3}}$ and $e' = \sqrt{3}$

$b^2 = a^2(1 - e^2)$

$b^2 = 6$

and $B^2 = A^2((e')^2 - 1) = (2) \Rightarrow B^2 = 2$

sum of LR $= \frac{2b^2}{a}+\frac{2B^2}{A} = 8$

17. If A and B are two events such that $P(A\cap B)=0.1$, and $P(A|B)$ and $P(B|A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})}$ is:

(1) $\frac{5}{3}$ (2) $\frac{4}{3}$

(3) $\frac{9}{4}$ (4) $\frac{7}{4}$

Ans. (3)

Sol. $12x^2 - 7x + 1 = 0$

$x = \frac{1}{3}, \frac{1}{4}$

Let $P\left(\frac{A}{B}\right)=\frac{1}{3}$ & $P\left(\frac{B}{A}\right)=\frac{1}{4}$

$\frac{P(A\cap B)}{P(B)}=\frac{1}{3}$ & $\frac{P(A\cap B)}{P(A)}=\frac{1}{4}$

$\Rightarrow P(B) = 0.3$

& $P(A) = 0.4$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.3 + 0.4 - 0.1 = 0.6$

Now $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})} = \frac{P(\overline{A\cap B})}{P(\overline{A\cup B})}$

$= \frac{1-P(A\cap B)}{1-P(A\cup B)} = \frac{1-0.1}{1-0.6} = \frac{9}{4}$

18. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos 2\theta$ and $2\cos^2\theta = 3\sin\theta$ is

(1) $\frac{\pi}{2}$ (2) 4π

(3) $\frac{5\pi}{6}$ (4) π

Ans. (4)

Sol. $2\sin^2\theta = \cos 2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(2\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

so common equation which satisfy both equations is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad (\theta \in [0, 2\pi])$

Sum $= \pi$

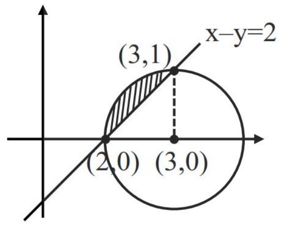
19. Let the curve $z(1 + i) + \bar{z}(1-i) = 4$, $z \in C$, divide the region $|z - 3| \le 1$ into two parts of areas α and β. Then $|\alpha - \beta|$ equals :

(1) $1+\frac{\pi}{2}$ (2) $1+\frac{\pi}{3}$

(3) $1+\frac{\pi}{4}$ (4) $1+\frac{\pi}{6}$

Ans. **(1)**

Sol.



Let $z = x + iy$

$(x + iy)(1 + i) + (x - iy)(1 - i) = 4$

$x + ix + iy - y + x - ix - iy - y = 4$

$2x - 2y = 4$

$x - y = 2$

$|z - 3| \le 1$

$(x - 3)^2 + y^2 \le 1$

Area of shaded region $= \frac{\pi.1^2}{4} - \frac{1}{2}.1.1 = \frac{\pi}{4} - \frac{1}{2}$

Area of unshaded region inside the circle

$= \frac{3}{4}\pi.1^2 + \frac{1}{2}.1.1 = \frac{3\pi}{4} + \frac{1}{2}$

$\therefore$ difference of area $= \left(\frac{3\pi}{4} + \frac{1}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{2} + 1$

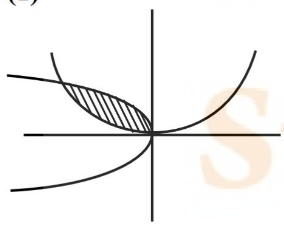
20. The area of the region enclosed by the curves $y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is :

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$

(3) 5 (4) 8

Ans. **(1)**

Sol.



$y = (x-2)^2$, $y^2 = 8(x - 2)$

$y = x^2$, $y^2 = -8x$

$= \frac{16ab}{3} = \frac{16 \times \frac{1}{4} \times 2}{3} = \frac{8}{3}$

SECTION-B

21. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^6+4x}{\sqrt{1-x^2}}$, $-1 < x < 1$ such that $f(0) = 0$. If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$ then α^2 is equal to __________.

Ans. **(27)**

Sol. I.F. $e^{-\frac{1}{2}\int \frac{2x}{1-x^2}dx} = e^{-\frac{1}{2}\ell n(1-x^2)} = \sqrt{1-x^2}$

$y \times \sqrt{1-x^2} = \int (x^6 + 4x)dx = \frac{x^7}{7} + 2x^2 + c$

Given $y(0) = 0 \Rightarrow c = 0$

$y = \dfrac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}$

Now, $6\int_{-\frac{1}{2}}^{\frac{1}{2}} \dfrac{\frac{x^7}{7} + 2x^2}{\sqrt{1-x^2}}dx = 6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}}dx$

$= 24\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$

Put $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$= 24\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{6}} \left(\frac{1-\cos 2\theta}{2}\right)d\theta = 12\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}}$

$= 12\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= 2\pi - 3\sqrt{3}$

$\alpha^2 = (3\sqrt{3})^2 = 27$

22. Let $A(6, 8)$, $B(10 \cos\alpha, -10 \sin\alpha)$ and $C(-10 \sin\alpha, 10 \cos\alpha)$, be the vertices of a triangle. If $L(a, 9)$ and $G(h, k)$ be its orthocenter and centroid respectively, then $(5a - 3h + 6k + 100 \sin 2\alpha)$ is equal to____________

Ans. **(145)**

Sol. All the three points A, B, C lie on the circle $x^2 + y^2 = 100$ so circumcentre is (0, 0)

$$\underset{O(0,0)}{\bullet}\overset{1}{\text{———}}\underset{G(h,k)}{+}\overset{2}{\text{———}}\underset{L(a,9)}{\bullet}$$

$$\frac{a+0}{3} = h \Rightarrow a = 3h$$

and $\dfrac{9+0}{3} = k \Rightarrow k = 3$

also centroid $\dfrac{6+10\cos\alpha - 10\sin\alpha}{3} = h$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3h - 6$ ....(i)

and $\dfrac{8+10\cos\alpha - 10\sin\alpha}{3} = k$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3k - 8 = 9 - 8 = 1$....(ii)

on squaring $100(1 - \sin2\alpha) = 1$

$\Rightarrow 100\sin2\alpha = 99$

from equ. (i) and (ii) we get $h = \dfrac{7}{3}$

Now $5a - 3h + 6k + 100\sin2\alpha$

$= 15h - 3h + 6k + 100\sin2\alpha$

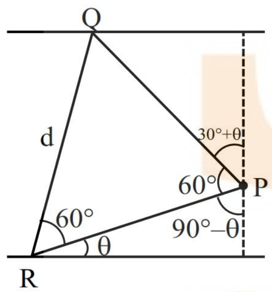
$= 12 \times \dfrac{7}{3} + 18 + 99$

$= 145$

23. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to ________.

Ans. **(28)**

Sol.



$PR = \text{cosec}\theta$, $PQ = 4\sec(30 + \theta)$

For equilateral

$d = PR = PQ$

$\Rightarrow \cos(\theta + 30°) = 4\sin\theta$

$\Rightarrow \dfrac{\sqrt{3}}{2}\cos\theta - \dfrac{1}{2}\sin\theta = 4\sin\theta$

$\Rightarrow \tan\theta = \dfrac{1}{3\sqrt{3}}$

$QR^2 = d^2 = \text{cosec}^2\theta = 28$

24. If $\displaystyle\sum_{r=1}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to ______.

Ans. **(465)**

Sol. $\displaystyle\sum_{r=1}^{30} \frac{r^2({}^{30}C_r)^2}{{}^{30}C_{r-1}}$

$$= \sum_{r=1}^{30} r^2\left(\frac{31-r}{r}\right)\cdot\frac{30!}{r!(30-r)!}$$

$$\left(\because \frac{{}^{30}C_r}{{}^{30}C_{r-1}} = \frac{30-r+1}{r} = \frac{31-r}{r}\right)$$

$$= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!(30-r)!}$$

$$= 30\sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!(30-r)!}$$

$$= 30\sum_{r=1}^{30} (30-r+1)\,{}^{29}C_{30-r}$$

$$= 30\left(\sum_{r=1}^{30}(31-r)\,{}^{29}C_{30-r} + \sum_{r=1}^{30} {}^{29}C_{30-r}\right)$$

$= 30(29 \times 2^{28} + 2^{29}) = 30(29 + 2)2^{28}$

$= 15 \times 31 \times 2^{29}$

$= 465(2^{29})$

$\alpha = 465$

25. Let A = {1, 2, 3}. The number of relations on A, containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is ______.

Ans. **(3)**

Sol. Transitivity

$(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$

For reflexive $(1, 1), (2, 2)\ (3, 3) \in R$

Now (2, 1), (3, 2), (3, 1)

(3, 1) cannot be taken

(1) (2, 1) taken and (3, 2) not taken

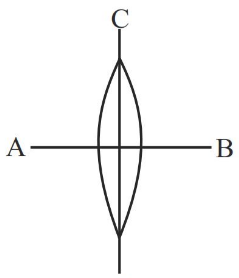
(2) (3, 2) taken and (2, 1) not taken

(3) Both not taken

therefore 3 relations are possible.

SECTION-A

26. A symmetric thin biconvex lens is cut into four equal parts by two planes AB and CD as shown in figure. If the power of original lens is 4D then the power of a part of the divided lens is

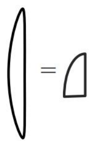


(1) 8D (2) 4D

(3) D (4) 2D

Ans. (4)

Sol.  $\frac{1}{f_1}=(\mu-1)\frac{2}{R}=P=4D$

 $\frac{1}{f_2}=(\mu-1)\frac{1}{R}=\frac{P}{2}=2D$

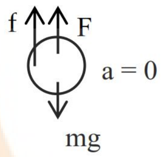
27. A small rigid spherical ball of mass M is dropped in a long vertical tube containing glycerine. The velocity of the ball becomes constant after some time. If the density of glycerine is half of the density of the ball, then the viscous force acting on the ball will be

(consider g as acceleration due to gravity)

(1) $\frac{3}{2}Mg$ (2) $\frac{Mg}{2}$

(3) Mg (4) 2 Mg

Ans. (2)

Sol. 

$mg - F_B - f = 0$

$\Rightarrow mg - \frac{mg}{2} - f = 0$

$\therefore f = \frac{mg}{2}$

28. The maximum percentage error in the measurment of density of a wire is

[Given, mass of wire = (0.60 ± 0.003)g

radius of wire = (0.50 ± 0.01)cm

length of wire (10.00 ± 0.05)cm]

(1) 4 (2) 5

(3) 8 (4) 7

Ans. (2)

Sol. $d=\frac{m}{\text{vol.}}=\frac{m}{\pi R^2\ell} \Rightarrow \frac{d\rho}{\rho}=\frac{dm}{m}+\frac{2dR}{R}+\frac{d\ell}{\ell}$

$\Rightarrow \frac{d\rho}{\rho}=\left(\frac{0.003}{0.6}+\frac{2\times0.01}{0.5}+\frac{0.05}{10}\right)100=5\%$

29. A series LCR circuit is connected to an alternating source of emf E. The current amplitude at resonant frequency is I_0. If the value of resistance R becomes twice of its initial value then amplitude of current at resonance will be

(1) I_0 (2) $\frac{I_0}{2}$

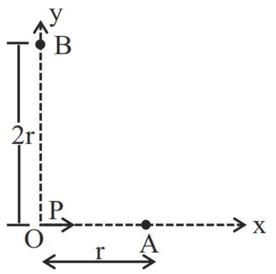
(3) $\frac{I_0}{\sqrt{2}}$ (4) $2I_0$

Ans. (2)

Sol. Initially, $I_0=\frac{\varepsilon_m}{R}$

Finally, $I_0^1=\frac{\varepsilon_m}{2R}=\frac{I_0}{2}$

30. For a short dipole placed at origin O, the dipole moment P is along x-axis, as shown in the figure. If the electric potential and electric field at A are V_0 and E_0, respectively, then the correct combination of the electric potential and electric field, respectively, at point B on the y-axis is given by



(1) $\frac{V_0}{2}$ and $\frac{E_0}{16}$ (2) zero and $\frac{E_0}{8}$

(3) zero and $\frac{E_0}{16}$ (4) V_0 and $\frac{E_0}{4}$

Ans. **(3)**

Sol. $E_A = \frac{2kP}{r^3} = E_0$ & $V_A = \frac{kP}{r^2} = v_0$

$E_B = \frac{kP}{(2r)^3} = \frac{E_0}{16}$ & $V_B = \frac{k\vec{P}.\hat{r}}{r^2} = 0$

31. Which one of the following is the correct dimensional formula for the capacitance in F ? M, L, T and C stand for unit of mass, length, time and charge,

(1) $[F] = [C^2M^{-2}L^2T^2]$

(2) $[F] = [CM^{-2}L^{-2}T^{-2}]$

(3) $[F] = [CM^{-1}L^{-2}T^2]$

(4) $[F] = [C^2M^{-1}L^{-2}T^2]$

Ans. **(4)**

Sol. $C = \frac{q}{V} = \frac{q.q}{V.q} = \frac{q^2}{WD} = \frac{C^2}{ML^2T^{-2}} = C^2M^{-1}L^{-2}T^2$

32. An electron projected perpendicular to a uniform magnetic field B moves in a circle. If Bohr's quantization is applicable, then the radius of the electronic orbit in the first excited state is :

(1) $\sqrt{\frac{2h}{\pi eB}}$ (2) $\sqrt{\frac{4h}{\pi eB}}$

(3) $\sqrt{\frac{h}{2\pi eB}}$ (4) $\sqrt{\frac{h}{\pi eB}}$

Ans. **(4)**

Sol. $r = \frac{mv}{eB}$ & $mvr = \frac{nh}{2\pi} \Rightarrow (eBr)r = \frac{nh}{2\pi}$

$\Rightarrow r = \sqrt{\frac{nh}{2\pi eB}}$

first excited state : $n = 2 \therefore r = \sqrt{\frac{h}{\pi eB}}$

33. For a diatomic gas, if $\gamma_1 = \left(\frac{Cp}{Cv}\right)$ for rigid molecules and $\gamma_2 = \left(\frac{Cp}{Cv}\right)$ for another diatomic molecules, but also having vibrational modes. Then, which one of the following options is correct ?

(Cp and Cv are specific heats of the gas at constant pressure and volume)

(1) $\gamma_2 > \gamma_1$ (2) $\gamma_2 = \gamma_1$

(3) $2\gamma_2 = \gamma_1$ (4) $\gamma_2 < \gamma_1$

Ans. **(4)**

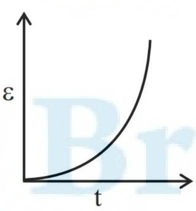
Sol. $\gamma = \frac{2}{f} + 1$

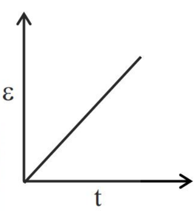
without vibration : $f = 5 : \gamma_1 = 1.4$

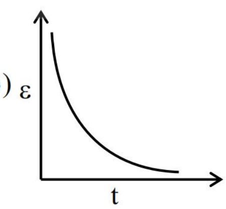
without vibration : $f = 7 : \gamma_2 = 1.14$

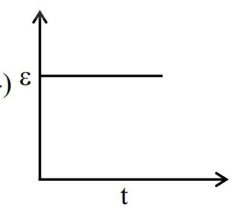
$\therefore \gamma_2 < \gamma_1$

34. A rectangular metallic loop is moving out of a uniform magnetic field region to a field free region with a constant speed. When the loop is partially inside the magnate field, the plot of magnitude of induced emf (ε) with time (t) is given by

(1) 

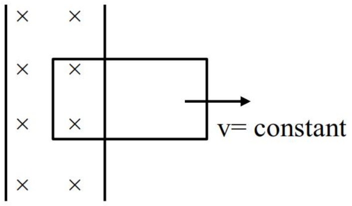
(2) 

(3) 

(4) 

Ans. **(4)**

Sol.



B = constant

Motional emf : $\varepsilon = B\ell v$ = constant

35. A light source of wavelength λ illuminates a metal surface and electrons are ejected with maximum kinetic energy of 2 eV. If the same surface is illuminated by a light source of wavelength $\frac{\lambda}{2}$, then the maximum kinetic energy of ejected electrons will be (The work function of metal is 1 eV)

(1) 2 eV (2) 6 eV

(3) 5 eV (4) 3 eV

Ans. (3)

Sol. $\frac{hc}{\lambda} = \phi + eV \Rightarrow \frac{hc}{\lambda} = 1 + 2 = 3eV$ ......(1)

$\frac{hc}{\lambda/2} = 6 = 1 + k_{max} \quad \therefore k_{max} = 5eV$

36. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : A simple pendulum is taken to a planet of mass and radius, 4 times and 2 times, respectively, than the Earth. The time period of the pendulum remains same on earth and the planet.

Reason (R) : The mass of the pendulum remains unchanged at Earth and the other planet. In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

(2) **(A)** is true but **(R)** is false

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Ans. (1)

Sol. $g = \frac{GM}{R^2}$

$g' = \frac{G(4M)}{(2R)^2} = g$

A is correct, R is correct ; but since $T = 2\pi\sqrt{\frac{\ell}{g}}$ doesn't depend on mass ; R doesn't explain A.

37. The torque due to the force $(2\hat{i} + \hat{j} + 2\hat{k})$ about the origin, acting on a particle whose position vector is $(\hat{i} + \hat{j} + \hat{k})$, would be

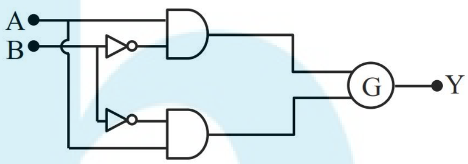
(1) $\hat{i} - \hat{j} + \hat{k}$ (2) $\hat{i} + \hat{k}$

(3) $\hat{i} - \hat{k}$ (4) $\hat{j} - \hat{k}$

Ans. (3)

Sol. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} - 0\hat{j} - \hat{k}$

38.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

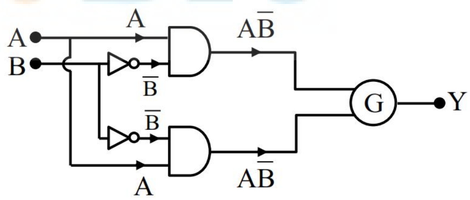
To obtain the given truth table, following logic gate should be placed at G :

(1) NOR Gate (2) AND Gate

(3) NAND Gate (4) OR Gate

NTA Ans. (1)

Sol.



For NOR gate : $\overline{A\bar{B}} = \bar{A} + B$

$$\therefore \text{Truth table} \quad \begin{array}{ccc} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$$

$\therefore$ Bonus

39. A force $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$ is applied on a particle and it undergoes a displacement $\hat{i} - 2\hat{j} - \hat{k}$. What will be the value of b, if work done on the particle is zero.

(1) 0 (2) $\frac{1}{2}$

(3) $\frac{1}{3}$ (4) 2

Ans. (2)

Sol. WD = $\vec{F}.\vec{S} = 2 - 2b - 1 = 0$

$\therefore b = \frac{1}{2}$

40. Given below are two statements. On is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : In Young's double slit experiment, the fringes produced by red light are closer as compared to those produced by blue light.

Reason (R) : The fringe width is directly proportional to the wavelength of light.

In the light of above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**.

(4) **(A)** is true but **(R)** is false.

Ans. (2)

Sol. $\beta = \frac{\lambda D}{d}$ & $\lambda_R > \lambda_b$

$\therefore \beta_R > \beta_b$

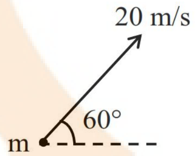
41. A ball of mass 100 g is projected with velocity 20 m/s at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is :

(1) 20 J (2) 15 J

(3) zero (4) 5 J

Ans. (2)

Sol.



$k_i = \frac{1}{2}mv^2$

$k_f = \frac{1}{2}m(v\cos 60°)^2 = \frac{1}{8}mv^2$

$\Delta k = k_i - k_f = \frac{3}{8}mv^2 = \frac{3}{8} \times 0.1 \times 400 = 15J$

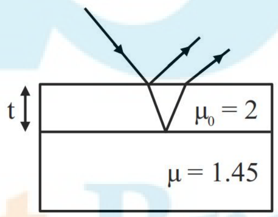
42. A transparent film of refractive index, 2.0 is coated on a glass slab of refractive index, 1.45. What is the minimum thickness of transparent film to be coated for the maximum transmission of Green light of wavelength 550 nm. [Assume that the light is incident nearly perpendicular to the glass surface.]

(1) 94.8 nm (2) 68.7 nm

(3) 137.5 nm (4) 275 nm

Ans. (3)

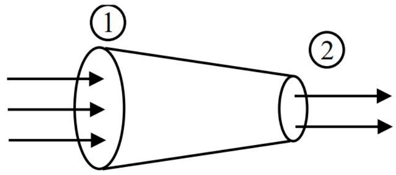
Sol.



For transmitted green light to be maxima, reflected green should be minima.

$\Delta P = 2\mu_0 t = n\lambda$

$\Rightarrow t = \frac{n\lambda}{2\mu_0} \therefore t_{min} = \frac{\lambda}{2\mu_0} = \frac{550}{2 \times 2} = 137.5$

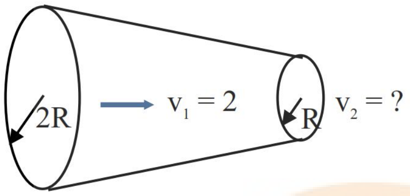
43. 

The tube of length L is shown in the figure. The radius of cross section at the point (1) is 2 cm and at the point (2) is 1 cm, respectively. If the velocity of water entering at point (1) is 2 m/s, then velocity of water leaving the point (2) will be :

(1) 2 m/s (2) 4 m/s

(3) 6 m/s (4) 8 m/s

Ans. **(4)**

Sol. 

$A_1v_1 = A_2V_2 \Rightarrow 2\pi(2R)^2 = V_2\pi R^2$

$\therefore V_2 = 8$ m/s

44. Given are statements for certain thermodynamic variables,

(A) Internal energy, volume (V) and mass (M) are extensive variables.

(B) Pressure (P), temperature (T) and density (ρ) are intensive variables.

(C) Volume (V), temperature (T) and density (ρ) are intensive variables.

(D) Mass (M), temperature (T) and internal energy are extensive variables.

Choose the **correct** answer from the points given below :

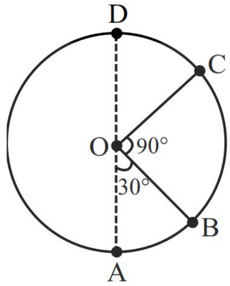
(1) (C) and (D) only (2) (D) and (A) only

(3) (A) and (B) only (4) (B) and (C) only

Ans. **(3)**

Sol. Extensive variables depends on size or mass of system ex : internal energy, volume, mass

45. A body of mass 100 g is moving in circular path of radius 2 m on vertical plane as shown in figure. The velocity of the body at point A is 10 m/s. The ratio of its kinetic energies at point B and C is :



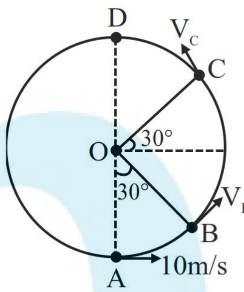
(Take acceleration due to gravity as 10 m/s^2)

(1) $\frac{2+\sqrt{3}}{3}$ (2) $\frac{2+\sqrt{2}}{3}$

(3) $\frac{3+\sqrt{3}}{2}$ (4) $\frac{3-\sqrt{2}}{2}$

Ans. **(3)**

Sol.



$$\frac{1}{2}m \times 100 + 0 = \frac{1}{2}mV_B^2 + mg\left(R - \frac{R\sqrt{3}}{2}\right)$$

$$100 = V_B^2 + 2gR\left(1 - \frac{\sqrt{3}}{2}\right]$$

$$V_B^2 = 100 - 20(2 - \sqrt{3})$$

$$V_B^2 = 60 + 20\sqrt{3})$$

$$K.E_B = \frac{1}{2}mV_B^2 = \frac{m}{2}(60 + 20\sqrt{3})$$

$$\frac{1}{2}m(100) = \frac{1}{2}mV_C^2 + mg\left(\frac{3R}{2}\right)$$

$$100 = V_C^{\,2} = 60$$

$$V_C^{\,2} = 40$$

$$K.E_C = \frac{1}{2}mV_C^2 = \frac{1}{2}m(40)$$

$$K.E_B = \frac{60 + 20\sqrt{3}}{40} = \frac{3}{2} + \frac{\sqrt{3}}{2} = \frac{3+\sqrt{3}}{2}$$

SECTION-B

46. A proton is moving undeflected in a region of crossed electric and magnetic fields at a constant speed of 2×10^5 ms^{-1}. When the electric field is switched off, the proton moves along a circular path of radius 2 cm. The magnitude of electric field is $x \times 10^4$ N/C. the value of x is _____.
Take the mass of the proton = 1.6×10^{-27} kg.

Ans. **(2)**

Sol. For uniform speed $V = \dfrac{E}{B}$

$$R = \frac{mV}{eB}$$

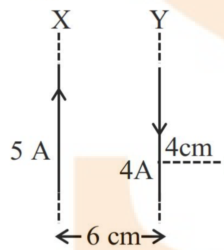
$$= \frac{mV^2}{eE}$$

$$\Rightarrow E = \frac{mV^2}{eR}$$

$$= \frac{1.6\times10^{-27}\times4\times10^{10}}{1.6\times10^{-19}\times2\times10^{-2}}$$

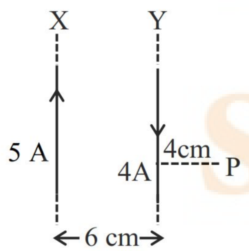
$= 2 \times 10^4$ N/C.

47. Two long parallel wires X and Y, separated by a distance of 6 cm, carry currents of 5A and 4A, respectively, in opposite directions as shown in the figure. Magnitude of the resultant magnetic field at point P at a distance of 4 cm from wire Y is $x \times 10^{-5}$ T. The value of x is __________. Take permeability of free space as $\mu_0 = 4\pi \times 10^{-7}$ SI units.



Ans. **(1)**

Sol.



$$B = \frac{\mu_0(5)}{2\pi\times.01} - \frac{\mu_0 4}{2\pi\times0.04}$$

$$= -\frac{100\mu_0}{4\pi}$$

$= -100 \times 10^{-7}$

$= -1 \times 10^{-5}$ T

48. A parallel plate capacitor of area A = 16 cm^2 and separation between the plates 10 cm, is charged by a DC current. Consider a hypothetical plane surface of area A_0 = 3.2 cm^2 inside the capacitor and parallel to the plates. At an instant, the current through the circuit is 6A. At the same instant the displacement current through A_0 is ________ mA.

Ans. **(1200)**

Sol. $J_d = \dfrac{I}{A} = \dfrac{6}{16}$

$\therefore$ I through small area = $J_d \times A' = \dfrac{6}{16} \times 3.2$ =

1.2A =1200 mA

49. A tube of length 1m is filled completely with an ideal liquid of mass 2M, and closed at both ends. The tube is rotated uniformly in horizontal plane about one of its ends. If the force exerted by the liquid at the other end is F then angular velocity of the tube is $\sqrt{\dfrac{F}{\alpha M}}$ in SI unit. The value of α is ______.

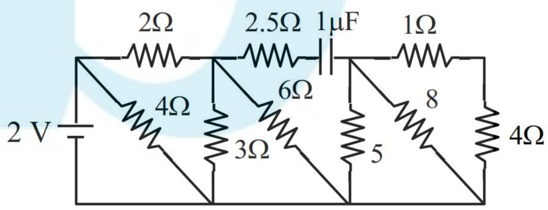
Ans. **(1)**

Sol.

$$F = 2M\omega^2 \frac{\ell}{2} = Mw^2\ell$$

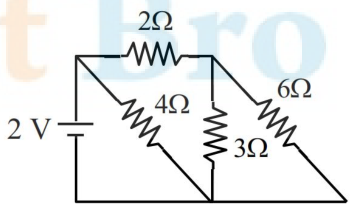
$$\omega = \sqrt{\frac{F}{M\ell}}$$

50. The net current flowing in the given circuit is ________ A.



Ans. **(1)**

Sol.



$R_{eq} = 2\Omega$

$I = \dfrac{2}{2} = 1$A

CHEMISTRY

SECTION-A

51. Arrange the following compounds in increasing order of their dipole moment :

HBr, H_2S, NF_3 and $CHCl_3$

(1) $NF_3 < HBr < H_2S < CHCl_3$

(2) $HBr < H_2S < NF_3 < CHCl_3$

(3) $H_2S < HBr < NF_3 < CHCl_3$

(4) $CHCl_3 < NF_3 < HBr < H_2S$

Ans. **(1)**

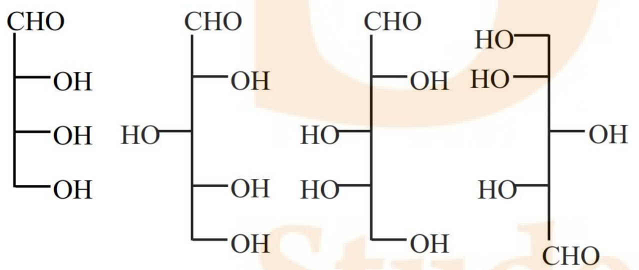
Sol. Increasing order of Dipole moment

$$NF_3 < HBr < H_2S < CHCl_3$$

$$\mu = 0.24D \quad 0.79D \quad 0.95D \quad 1.04D$$

It is NCERT Data Based

52. Identify the number of structure/s from the following which can be correlated to D-glyceraldehyde.



(A) (B) (C) (D)

(1) three

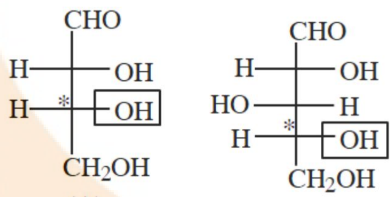
(2) two

(3) four

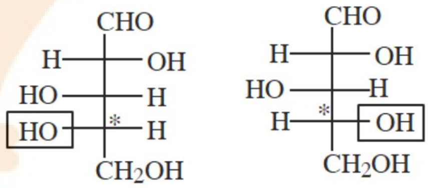
(4) one

Ans. **(1)**

Sol.



(A) (B)



(C) (D)

In A, B, D – OH group in right hand side then D-configuration is assigned

53. The maximum covalency of a non-metallic group 15 element 'E' with weakest E–E bond is :

(1) 5

(2) 3

(3) 6

(4) 4

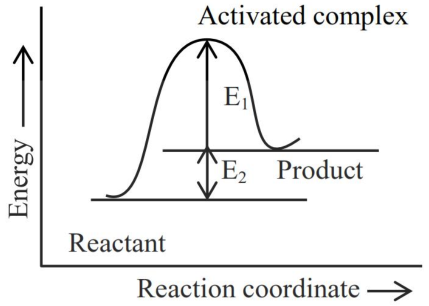
Ans. **(4)**

Sol. N – N < P – P : single (σ) bond strength

Due to L.P.-L.P. replusion

and maximum possible covalency of nitrogen is 4.

54. Consider the given figure and choose the **correct** option :



(1) Activation energy of backward reaction is E_1 and product is more stable than reactant.

(2) Activation energy of forward reaction is $E_1 + E_2$ and product is more stable than reactant.

(3) Activation energy of forward reaction is $E_1 + E_2$ and product is less stable than reactant.

(4) Activation energy of both forward and backward reaction is $E_1 + E_2$ and reactant is more stable than product.

Ans. **(3)**

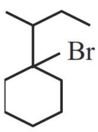
Sol. Activation energy of forward reaction = $E_1 + E_2$

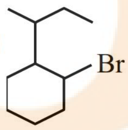
Energy of product > Energy of reactant

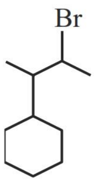
Stability

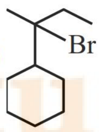
Reactant > Product

55. When sec-butylcyclohexane reacts with bromine in the presence of sunlight, the major product is :

(1) 

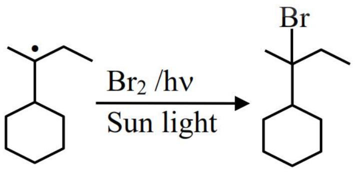
(2) 

(3) 

(4) 

Ans. **(4)**

Sol.



Formation of more stable free radical intermediate

56. The species which does not undergo disproportionation reaction is :

(1) ClO_2^- (2) ClO_4^-

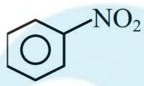
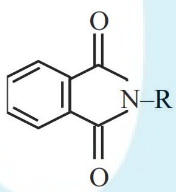
(3) ClO^- (4) ClO_3^-

Ans. **(2)**

Sol. $ClO_4^- \rightarrow x + \{(-2) \times 4\} = -1 \Rightarrow x = +7$

Chlorine is in its maximum oxidation state, so disproportionation not possible in ClO_4^-.

57. Match the Compounds **(List-I)** with the appropriate Catalyst/Reagents **(List-II)** for their reduction into corresponding amines.

	List-I **(Compounds)**		**List-II** **(Catalyst/Reagents)**
(A)	$R-\overset{\overset{O}{\parallel}}{C}-NH_2$	(I)	NaOH (aqueous)
(B)		(II)	H_2/Ni
(C)	$R-C\equiv N$	(III)	$LiAlH_4$, H_2O
(D)		(IV)	Sn, HCl

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(2) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

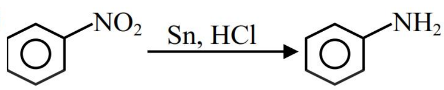
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

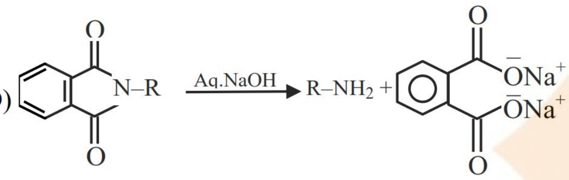
Ans. **(4)**

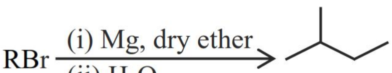
Shift

Sol. (A) $R-\overset{\overset{\displaystyle O}{||}}{C}-NH_2 \xrightarrow[H_2O]{LiAlH_4} R-CH_2-NH_2$

(B) 

(C) $R-C\equiv N \xrightarrow{H_2/Ni} R-CH_2NH_2$

(D) 

58. 

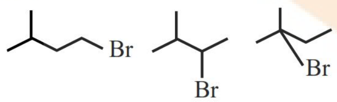
2-Methylbutane

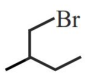
The maximum number of RBr producing 2-methylbutane by above sequence of reactions is _____. (Consider the structural isomers only)

(1) 4 (2) 5

(3) 3 (4) 1

Ans. **(1)**



Sol.  Total number of possible structures of RBr = 4

59. Match **List-I** with **List-II**.

	List-I (Partial Derivatives)		**List-II (Thermodynamic Quantity)**
(A)	$\left(\frac{\partial G}{\partial T}\right)_P$	(I)	C_P
(B)	$\left(\frac{\partial H}{\partial T}\right)_P$	(II)	–S
(C)	$\left(\frac{\partial G}{\partial P}\right)_T$	(III)	C_V
(D)	$\left(\frac{\partial U}{\partial T}\right)_V$	(IV)	V

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Ans. **(2)**

Sol. (A) $dG = VdP - SdT$

Constant pressure

$dG = -SdT$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

(B) $dH = (dq)_P = nC_PdT$

$$\left(\frac{\partial H}{\partial T}\right)_P = C_P$$

(C) $dG = VdP - SdT$

At constant temperature

$dG = VdP$

$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

(D) $dU = nC_VdT = (q)_V$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V$$

60. The correct order of the following complexes in terms of their crystal field stabilization energies is :

(1) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+}$

(2) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

(3) $[Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_4]^{2+} < [Co(en)_3]^{3+}$

(4) $[Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_4]^{2+}$

Ans. **(2)**

Sol. Order of CFSE

$$[Co(NH_3)_4]^{+2} < \underbrace{[Co(NH_3)_6]^{+2} < [Co(NH_3)_6]^{+3}}_{\text{Charge}\uparrow\ \text{CFSE}\uparrow} \!\!\!\!\underbrace{ < [Co(en)_3]^{+3}}_{\text{On increasing strength of ligand CFSE}\uparrow}$$

SFL : NH_3 < en

61. Density of 3 M NaCl solution is 1.25 g/mL. The molality of the solution is :

(1) 1.79 m (2) 2 m

(3) 3 m (4) 2.79 m

Ans. **(4)**

Sol. 3M NaCl, $d_{sol} = 1.25$ gm/mol

$$\text{Molality} = \frac{M \times 1000}{1000d - M \times M_W}$$

$$= \frac{3000}{1250 - 175.5} = 2.79$$

62. The molar solubility(s) of zirconium phosphate with molecular formula $(Zr^{4+})_3 \left(PO_4^{3-}\right)_4$ is given by relation :

(1) $\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$ (2) $\left(\frac{K_{sp}}{5348}\right)^{\frac{1}{6}}$

(3) $\left(\frac{K_{sp}}{8435}\right)^{\frac{1}{7}}$ (4) $\left(\frac{K_{sp}}{9612}\right)^{\frac{1}{3}}$

Ans. **(1)**

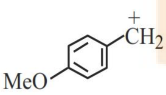
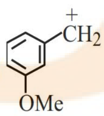
Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{+4}(aq) + 4PO_4^{-3}(aq)$

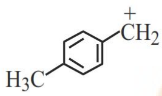
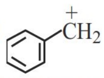
$$- \qquad 3s \qquad 4s$$

$K_{sp} = (3s)^3 (4s)^4 = 6912\, s^7$

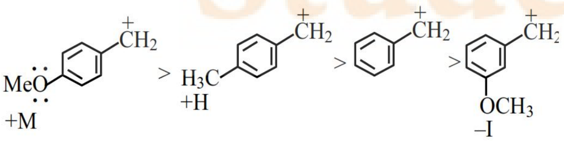
$$s = \left(\frac{K_{sp}}{6912}\right)^{1/7}$$

63. The most stable carbocation from the following is :

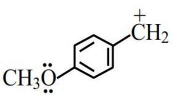
(1)  (2) 

(3)  (4) 

Ans. **(1)**

Sol. 

Due to +M effect of –OMe at para position

 Will be most stable among these

64. Given below are two statements :

Statement (I) : An element in the extreme left of the periodic table forms acidic oxides.

Statement (II) : Acid is formed during the reaction between water and oxide of a reactive element present in the extreme right of the periodic table.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement-I** is false but **Statement-II** is true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) Both **Statement-I** and **Statement-II** are true.

Ans. **(1)**

Sol. Statement-I : False but Statement-II is true.

On moving left to right in periodic table non-metallic character increases and we know that non-metal oxides are acidc in nature.

Non metallic character ↑ Acidic strength of oxide↑

65. Given below are two statements :

Statement (I) : A spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition.

Statement (II) : $2p_x$ and $2p_y$ are degenerate orbitals.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. **(4)**

Sol. No spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition because $2p_x$ and $2p_y$ orbitals are degenerate orbitals.

66. Given below are two statements :

Statement (I) : Nitrogen, sulphur, halogen and phosphorus present in an organic compound are detected by Lassaigne's Test.

Statement (II) : The elements present in the compound are converted from covalent form into ionic form by fusing the compound with Magnesium in Lassaigne's test.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Ans. (3)

Sol. The elements present in the compound are converted from covalent form into ionic form by fusing the compound with sodium in Lassigne's test

67. Identify the homoleptic complex(es) that is/are low spin.

(A) $[Fe(CN)_5NO]^{2-}$ (B) $[CoF_6]^{3-}$
(C) $[Fe(CN)_6]^{4-}$ (D) $[Co(NH_3)_6]^{3+}$
(E) $[Cr(H_2O)_6]^{2+}$

Choose the **correct** answer from the options given below :

(1) (B) and (E) only (2) (A) and (C) only
(3) (C) and (D) only (4) (C) only

Ans. (3)

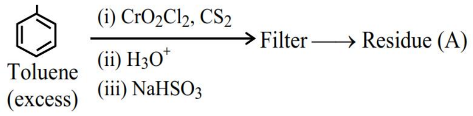
Sol. (A) $[Fe(CN)_5NO]^{-2}$ → Heteroleptic, Fe^{+2}, $3d^6$, $t_{2g}^6 e_g^0$, d^2sp^3, Low spin (3d series + SFL)

(B) $[CoF_6]^{-3}$ → Homoleptic, sp^3d^2, High spin, Co^{+3}, $3d^6$ (3d series + WFL)

(C) $[Fe(CN)_6]^{-4}$ → Homoleptic
Fe^{+2}, $3d^6$, d^2sp^3, t_{2g}^6 eg^0 Low spin
(3d series + SFL)

(D) $[Co(NH_3)_6]^{+3}$ → Homoleptic, Co^{+3} $3d^6$, d^2sp^3, t_{2g}^6 eg^0, Low spin (3d series + SFL)

(E) $[Cr(H_2O)_6]^{+2}$ → Homoleptic
Cr^{+2} $3d^4$, d^2sp^3, High spin t_{2g}^3 e_g^1
(3d series + WFL)

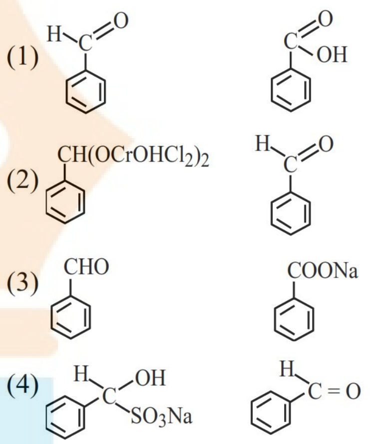
68. 

Residue (A) + HCl (dil.) → Compound (B)

Structure of residue (A) and compound (B)

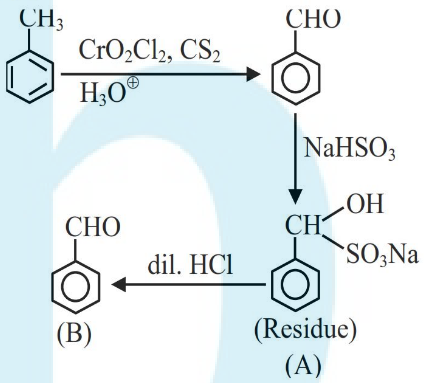
Formed respectively is :

[A] **[B]**



Ans. (4)

Sol.



69. Given below are two statements :

Statement (I) : Corrosion is an electrochemical phenomenon in which pure metal acts as an anode and impure metal as a cathode.

Statement (II) : The rate of corrosion is more in alkaline medium than in acidic medium.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true
(4) Statement I is true but Statement II is false

Ans. (4)

Sol. **Statement I :**

Corrosion is an example of electrochemical phenomenon

In which pure metal act as anode and impure metal (rusted metal) act as cathode.

Statement II :

Corrosion is more favourable in acid medium than alkaline so rate of corrosion is high is acid medium then alkaline.

70. The alkane from below having two secondary hydrogens is :

(1) 4-Ethyl-3,4-dimethyloctane

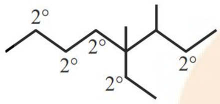
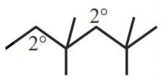
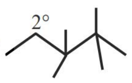
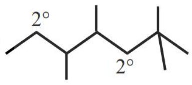
(2) 2,2,4,4-Tetramethylhexane

(3) 2,2,3,3-Tetramethylpentane

(4) 2,2,4,5-Tetramethylheptane

Ans. **(3)**

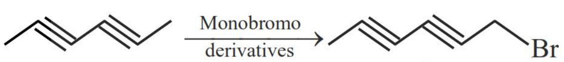
Sol.

Alkane	2°H
1. 	10
2. 	4
3. 	2
4. 	4

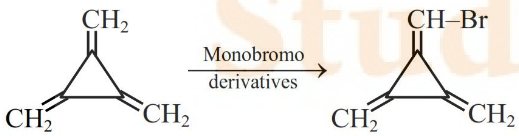
SECTION-B

71. The compound with molecular formula C_6H_6, which gives only one monobromo derivative and takes up four moles of hydrogen per mole for complete hydrogenation has _______ π electrons.

Ans. **(8)**

Sol. 

No. of π e^- = 8



N. of π e^- = 6

72. Niobium (Nb) and ruthenium (Ru) have "x" and "y" number of electrons in their respective 4d orbitals. The value of x + y is _______

Ans. **(11)**

Sol. $Z = 41 \rightarrow$ Nb (Niobium) : $[Kr]_{36}\,4d^4\,5s^1$

Number of electron in 4d = 4 = x

$Z = 44 \rightarrow$ Ru (Ruthenium) $[Kr]_{36}\,4d^7\,5s^1$

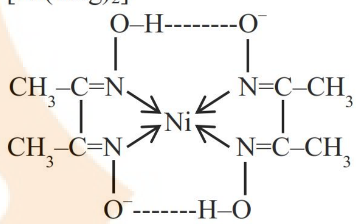
Number of electron in 4d = 7 = y

x + y = 11

73. The complex of Ni^{2+} ion and dimethyl glyoxime contains _______ number of Hydrogen (H) atoms.

Ans. **(14)**

Sol. $[Ni(dmg)_2]$



Number of H-atom = 14

74. Consider the following cases of standard enthalpy of reaction $\left(\Delta H_r^\circ \text{ in kJ mol}^{-1}\right)$

$$C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)\ \Delta H_1^\circ = -1550$$

$$C(\text{graphite}) + O_2(g) \rightarrow CO_2(g)\ \Delta H_2^\circ = -393.5$$

$$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(\ell)\ \Delta H_3^\circ = -286$$

The magnitude of $\Delta H^\circ_{f\ C_2H_6(g)}$ is _______ kJ mol^{-1} (Nearest integer).

Ans. **(95)**

Sol. $2C_{(\text{graphite})} + 3H_2(g) \rightarrow C_2H_6(g) \quad \Delta H_f = ?$

$C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta H_1 = -1550$

$C_{(\text{graphite})} + O_2(g) \rightarrow CO_2(g) \quad \Delta H_2 = -393.5$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l) \quad \Delta H_3 = -286$

$\Delta H_f = 2\Delta H_2 + 3\Delta H_3 - \Delta H_1$

$= 95$ kJ/mole.

75. 20 mL of 2 M NaOH solution is added to 400 mL of 0.5 M NaOH solution. The final concentration of the solution is _______ $\times 10^{-2}$ M. (Nearest integer).

Ans. **(57)**

Sol. $M_F = \dfrac{M_1V_1 + M_2V_2}{V_1 + V_2}$

$= \dfrac{2 \times 20 + 0.5 \times 400}{420} = 0.571$ M

$= 57.1 \times 10^{-2}$ M

$= 57$